Examples goodmodels

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1 Genus 3

The code for prime fields $\mathbb{F}_p$ is loaded as follows:

```
load "goodmodels_p.m";
```

Note that the package `pcc_p` from [1, 2], that we use for computing the zeta function, comes with `goodmodels` and is automatically loaded.

As an example, we generate a random quartic `f` in 3 variables over the finite field $\mathbb{F}_{97}$ and compute the best lift `Q` to characteristic 0 of the corresponding curve using the methods explained above (including all the optimizations).

```
p:=97;
f:=random_genus3(p);
Q:=optimal_model_genus3(f);
```

Now we compute the numerator of the zeta function of the curve defined by `f` by calling the function `num_zeta` from `pcc_p`:

```
chi:=num_zeta(Q,p);
```

Note that the denominator of the zeta function of a curve is always `(1-T)(1-pT)`, so the zeta function can be easily deduced from its numerator `chi`. To repeat this example over the non-prime field $\mathbb{F}_{3^{10}}$, the commands are as follows:

```
load "goodmodels_q.m";
q:=3^10;
f:=random_genus3(q);
Q:=optimal_model_genus3(f);
chi:=num_zeta(Q,q);
```

Alternatively, in both examples we can compute the zeta function of the curve defined by `f` using a single command, which takes care of the optimizations implicitly:

```
zeta:=zeta_genus3(f);
```

2 Genus 4

Again we load the code for a prime field $\mathbb{F}_p$:

```
load "goodmodels_p.m";
```

This time we generate a random quadric S2 of given discriminant and a random cubic S3 in 4 variables over the finite field $\mathbb{F}_{97}$ and compute the best lift Q to characteristic 0 of the corresponding curve using the methods explained above (including all the optimizations):

```
p:=97;
S2,S3:=random_genus4(p,0);
Q:=optimal_model_genus4(S2,S3);
```

Note that here we have taken χ_2 to be 0, for the other cases the second argument of random_genus4 should be set to 1 or -1. Now we compute the numerator of the zeta function of the curve defined by S2,S3 by calling the function num_zeta from pcc_p:

```
chi:=num_zeta(Q,p);
```

To repeat this example over the field $\mathbb{F}_{3^{10}}$, the commands are as follows:

```
load "goodmodels_q.m";
q:=3^10;
S2,S3:=random_genus4(q,0);
Q:=optimal_model_genus4(S2,S3);
chi:=num_zeta(Q,q);
```

For $\chi_2 = -1$, it is sometimes more efficient to replace the last two lines by:

```
Q,W0,Winf:=optimal_model_genus4(S2,S3:alternative_Winf:=true);
chi:=num_zeta(Q,q:W0:=W0,Winf:=Winf);
```

Alternatively, in both examples we can again compute the zeta function of the curve defined by S2,S3 using a single command:

```
zeta:=zeta_genus4(S2,S3);
```

This function automatically selects (what it expects to be) the most efficient of the two options for the last two lines above.

3 Genus 5

Trigonal case To compute the numerator chi of the zeta function of a random trigonal curve of genus 5 over $\mathbb{F}_{97}$ we type:

```
load "goodmodels_p.m";
p:=97;
S21,S22,S23,S31,S32:=random_genus5_trigonal(p);
Q:=optimal_model_genus5_trigonal(S21,S22,S23,S31,S32);
chi:=num_zeta(Q,p);
```

Note that S21,S22,S23 are the quadrics and S31,S32 the cubics defining the canonical ideal.

To repeat this example over the field $\mathbb{F}_{3^{10}}$ the commands are as follows:

```
load "goodmodels_q.m";
q:=3^10;
S21,S22,S23,S31,S32:=random_genus5_trigonal(q);
Q:=optimal_model_genus5_trigonal(S21,S22,S23,S31,S32);
chi:=num_zeta(Q,q);
```

Alternatively, in both examples we can again compute the zeta function of the curve defined by S21,S22,S23,S31,S32 using a single command:

```
zeta:=zeta_genus5_trigonal(S21,S22,S23,S31,S32);
```

Non-trigonal case To compute the numerator chi of the zeta function of a non-trigonal curve of genus 5 over $\mathbb{F}_{97}$ the commands are as follows:

```
load "goodmodels_p.m";
p:=97;
S21,S22,S23:=random_genus5_nontrigonal(p);
Q:=optimal_model_genus5_trigonal(S21,S22,S23);
chi:=num_zeta(Q,p);
```

To repeat this example over $\mathbb{F}_{3^{10}}$, we type:

```
load "goodmodels_q.m";
q:=3^10;
S21,S22,S23:=random_genus5_nontrigonal(q);
Q:=optimal_model_genus5_trigonal(S21,S22,S23);
chi:=num_zeta(Q,q);
```

It is sometimes more efficient to replace the last two lines by:

```
Q,W0,Winf:=optimal_model_genus5_trigonal(S21,S22,S23:alternative_Winf:=true);
chi:=num_zeta(Q,q:W0:=W0,Winf:=Winf);
```

Alternatively, in both examples we can again compute the zeta function of the curve defined by S21,S22,S23 using a single command:

```
zeta:=zeta_genus5_nontrigonal(S21,S22,S23);
```

This function automatically selects (what it expects to be) the most efficient of the two options for the last two lines above.

References

[1] J. Tuitman, *Counting points on curves using a map to* $\mathbb{P}^1$, Mathematics of Computation **85**, pp. 961-981 (2016)

[2] J. Tuitman, *Counting points on curves using a map to* $\mathbb{P}^1$*, II*, Finite Fields and Their Applications **45**, pp. 301-322 (2017)